

## Evidence for nonlinear flux diffusion from magnetization relaxation in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ single crystals

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Using a sensitive Hall probe, the magnetization relaxation of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  single crystals has been measured accurately. The sample was cooled down from  $T > T_c$  in a large bias magnetic field and then relaxation has been registered after a small change of the magnetic field. The effects of the bias field value, ranging from almost zero up to  $H_{\text{irrev}}$  ( $\approx 20$  kOe), and of the amplitude of the field change were studied at  $T = 20$  K. We find that the results are well described by the model for the self-organized critical state: there are short- and long-time domains of the relaxation, both characterized by a power-law dependence on time. The crossover time depends on the bias field (i.e. on the critical current density) and on the amplitude of field change. The long-time relaxation is independent of the initial field change.

### 1. Introduction

The dissipative processes occurring in high- $T_c$  superconductors for current densities exceeding their critical value  $j_c$  are of interest both in view of possible applications and as a general topic in type-II superconductivity. These processes can be best monitored by studies of the relaxation of magnetization after an abrupt change of the external field to which the superconducting sample is exposed.

In conventional type-II superconductors the Anderson–Kim model of flux bundles, hopping over potential barriers of height  $U_c$ , has been introduced to explain the magnetization relaxation [1]. The early experimental results confirmed the predicted logarithmic time dependence,

$$M \sim \frac{k_B T}{U_c} \ln \left( 1 + \frac{t}{\tau} \right), \quad (1)$$

as well as the linear increase of the relaxation rate  $S$  ( $= dM/d \ln t$ ) with temperature for  $t \gg \tau$  and  $k_B T \ll U_c$ , where  $\tau$  is related to the microscopic attempt time. In high- $T_c$  materials much larger values of the ratio  $k_B T/U_c$  can be achieved, so that the relaxation can be significantly faster. In fact, soon after

the discovery of oxide superconductors, it became clear that the relaxation in these materials is not logarithmic in time. Moreover, it was found that departure from the logarithmic time-dependence is not specific to high- $T_c$  materials. In the isotropic type-II superconductor  $\text{PbMo}_6\text{S}_8$  with  $T_c = 14.5$  K, the relaxation measurements give strongly field-dependent results. At low fields, a nearly logarithmic relaxation is observed, whereas at high fields a more complex behavior, described by the equation

$$M \sim \left( \frac{k_B T}{U_c} \ln \left( 1 + \frac{t}{\tau} \right) \right)^{-1/\mu}, \quad (2)$$

with  $\mu < 1$  is obtained [2]. Similar observations were made on  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  [3] and on other high- $T_c$  superconductors.

The first attempts to explain the non-logarithmic relaxation used the Ansatz of a superposition of independent relaxation processes with different pinning potentials  $U_c$  [4]. Detailed techniques have been developed in order to deconvolute the distribution of pinning energies from  $M(t)$ . Whereas it is possible to find an accurate fitting of the data using a proper distribution function, the principle of superposition of separate events is not likely to be the right

approach. There is an interesting parallel with the development of concepts in the theories of spin-glasses, where initially similar methods were used.

A time dependence that is characterized by a power of the logarithm as given by eq. (2), has been often invoked and is expected in the vortex-glass state [5]. The theory of collective flux creep [6] also yields an expression for  $M(t)$  as given by eq. (2) for  $t \gg \tau$ , provided that the potential barrier height is proportional to  $(j/j_c)^{-\mu}$ . Feigel'man et al. [6] have suggested the interpolation

$$M \sim \left( 1 + \mu \frac{k_B T}{U_c} \ln \left( 1 + \frac{t}{\tau} \right) \right)^{-1/\mu}, \quad (3)$$

which gives the result of the Anderson–Kim model for  $t \ll \tau$  and eq. (2) for  $t \gg \tau$ . For  $\mu \ll 1$ , which is the case for e.g. single-vortex creep, the condition  $(k_B T/U_c) \ln(1+t/\tau) \ll \mu^{-1}$  is easily satisfied. In that case, eq. (3) can be approximated by

$$M \sim (1+t/\tau)^{-k_B T/U_c}, \quad (4)$$

a relationship, which is also often used to fit experimental data [7,8].

Recently, an attempt has been made by Vinokur et al. [9] to find an analytical, approximate solution for the set of Maxwell equations describing the flux penetration into a thin slab with flux pinning barriers that grow logarithmically with decreasing current, according to

$$U(j) = U_c \ln(j_c/j). \quad (5)$$

Such a dependence of the pinning potential on current density leads to a nonlinear diffusion equation for the flux density. Some of the conclusions drawn in ref. [9] have been criticized [10], but the qualitative features of the results were confirmed by the exact numerical calculations of van der Beek et al. [11] and Schnack et al. [12]. In particular, it has been established that the pinning potential (5) leads indeed to self-organized criticality (SOC), a concept introduced by Bak et al. [13] in connection with the dynamics of highly dissipative, nonlinear systems.

The idea of SOC has attracted much attention in recent years [14–18], due to its universal character and multiple applications, of which the ones in the physical sciences form only a small branch of a broad spectrum [13]. It describes many-body systems

which, starting from an arbitrary initial state, organize themselves in a unique dynamical state which is critical in the sense that there exists no characteristic length or time scale. The concept proposed by Bak et al. is that this state is truly a critical point of the system (an attractor of an open dissipative system).

Recently this theory has been applied to high- $T_c$  materials as well. It was investigated numerically in the context of vortex dynamics in superconductors by Pla and Nori [15], used for the analysis of noise measurements [17,19] and recognized in the interpretation of magnetization relaxation [20]. It predicts certain features of the magnetic flux jumps, recently studied by Gerber et al. [21].

In the nonlinear flux diffusion theory, the SOC is reached after a certain time, not exactly known, from the moment the external field is changed. The wave of the flux profile spreads from the superconductor surface, into its centre. When the wave profile reaches the sample centre at the moment  $t^*$  (fig. 1), where  $t^*$  depends on  $j_c$  and on the initial field change  $\Delta H$ , its dynamics must change as it is no more free to propagate: a boundary condition must be imposed for the current at the sample centre  $j(0) = 0$ . Computer numerical solutions of the differential equations and Monte Carlo simulations [11,12] of the vortex dynamics show that the nonlinear flux-dif-

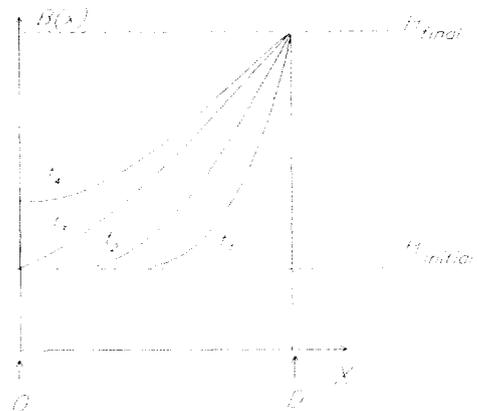


Fig. 1. Schematic evolution of the magnetic induction profile in the sample volume (0 means the centre of the sample of thickness  $2D$ ) after the external field is changed at  $t=0$  from the value  $H_{\text{initial}}$  to  $H_{\text{final}}$ . It is assumed that the magnetic field is significantly higher than  $H_{c1}$  and that the sample is cooled down in the field  $H_{\text{initial}}$  that was applied above  $T_c$ . The curves represent the flux profile for increasing time,  $t_1 < t_2 < t_3 = t^* < t_4$ .

fusion equations have solutions that are rather weakly dependent on the precise functional relationship between the activation energy and the current density and the magnetic induction.

There is a direct analogy between the formulation of the flux diffusion theory based on the assumption of a pinning potential as given by eq. (5) and that based on a nonlinear dependence of the conductivity on the electric field [22]. A nonlinear  $E$ - $j$  dependence can describe magnetization relaxation data in a very broad range of fields and temperatures in the case of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  [23–26] and can be considered as an alternative approach towards understanding dynamical phenomena in superconductors, especially in case of the Kosterlitz–Thouless transition, and of the magnetic properties in the temperature region near  $T_c$  or close to the irreversibility line [27].

The experimental implications of the nonlinear flux diffusion model are the following [9].

(1) There exists a characteristic time  $t^*(\Delta H/H, j_c(H))$  separating the short-time relaxation from the long-time one:  $t^* \rightarrow \infty$  for  $d_0 \rightarrow 0$ , and  $t^* \rightarrow 0$  when  $d_0$  becomes larger than the sample size, where  $d_0 = c\Delta H/4\pi j_c$  is the Bean penetration depth;

(2) In both time-domains, a power-law dependence should be observed. The approximate predictions for the time-dependent magnetization are given by [9,27]

$$4\pi M(t) = \left[ \frac{\alpha}{2} \left( \frac{\tau_0 + t}{\tau_0 + t^*} \right)^p - 1 \right] \Delta H \quad \text{for } t < t^*, \quad (6)$$

$$4\pi M(t) = -\Delta H \left( 1 - \frac{\alpha}{2} \left( \frac{\tau_0 + t}{\tau_0 + t^*} \right)^{-q} \right) \quad \text{for } t > t^*.$$

(3) One should note the relation between the exponents  $p = (1 - \alpha)/(2 - \alpha)$  and  $q = (1 - \alpha)/\alpha$ , characterizing the short-time and long-time relaxation, respectively:  $p \approx q$  for  $p, q \ll 1$ , whereas  $p \rightarrow 0.5$  for  $q$  increasing to values much larger than 1;

(4) the long-time relaxation is independent of the initial field change  $\Delta H$ , which follows from eq. (6), by substituting  $t^*$  by an expression in  $\Delta H$  [9,27].

The parameter  $\alpha$  in eq. (6) is related to the ratio of the flux-pinning energy and the average thermal energy,  $\epsilon = U_c/k_B T$ :

$$\alpha = \frac{\epsilon}{1 + \epsilon}, \quad (7)$$

in case the equations are based on the relation in eq. (5). If the discussion is based on a nonlinear current–voltage dependence [27],  $\alpha$  specifies the electric field dependence of the conductivity:

$$J(E) = \sigma(E)E, \quad \sigma \sim E^{-\alpha}. \quad (8)$$

In this paper, we present the first experimental verifications of the above mentioned predictions, given by eq. (6). The results were obtained on a  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  single crystal, with the external magnetic field oriented along the  $c$ -axis. In general, we can rule out the logarithmic time dependence as given in eq. (1). Instead, a power-law time dependence described above follows from our measurements. The dependence of the exponents  $p$  and  $q$  on various parameters gives confidence in the approximate validity of this theory. The high accuracy and the long-time stability which are required to distinguish between the various possible time dependences were achieved by the use of the Hall probe and a superconducting magnet in its persistent mode. This method is particularly useful for relaxation measurements and gained popularity in recent years, especially due to the successful efforts of Konczykowski et al. [28]. It has been applied in studies of the flux distribution on the sample surface, supporting qualitatively the predictions of the critical state model [29], as well as in attempts to investigate the time evolution of the flux distribution profile [7], resulting in a qualitative agreement with the predictions of Vinokur et al. [9].

## 2. Experimental details

Magnetization and magnetic relaxation measurements were performed on three single-crystalline  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  samples, characterized by sharp superconducting transitions in the AC susceptibility. The analysis of  $M(H)$  and  $M(T)$  results on these samples has been reported elsewhere [30]. In this paper we present a series of measurements performed on one of these samples, which has a circular disk shape with a diameter of 2.4 mm and a thickness of 0.2 mm. Magnetization and AC susceptibility measurements yielded a  $T_c$  of about 90 K. The crystallographic  $c$ -axis is perpendicular to the large sur-

face of the sample and this was the direction of the applied magnetic field.

A commercial Hall probe (supplied by Lake Shore Cryotronics, Inc.) is used as the sensor of the magnetic field at the surface of the magnetized sample. Its sensitivity is  $8 \mu\Omega/\text{G}$ . With the Linear Research resistance bridge LR-400, an excellent stability, linearity of the response to the field and an extremely small temperature dependence of the background signal are obtained. Under certain conditions, we can reach a resolution of about 10 mG. The field produced by the Hall probe at the sample surface is lower than about 10 mG for the injection current of 3 mA, that is typically used. The sensitive area of the Hall probe (1 mm in diameter) is comparable to the sample size and the field-sensitive region is placed at a distance of about 0.5 mm from the sample surface.

The measured stray field turns out to be linearly related to  $M$ , the magnetization of the sample. This has been verified in low fields and at low temperatures by measuring the virgin magnetization curve in the Meissner state. In this case, the measured magnetization,  $4\pi M_{\text{exp}} = H_s - H$ , where  $H_s$  is the field registered by the Hall probe and  $H$  is the externally applied field, should be found to reflect perfect diamagnetism, i.e.  $4\pi M = -H$ . We found  $4\pi M_{\text{exp}} = -0.39H$ , indicating that in the given geometry the sample does not screen the field perfectly. We have used the result of this calibration,  $4\pi M = 4\pi M_{\text{exp}}/0.39 = (H_s - H)/0.39$ , at all fields, although its validity for high fields depends on certain assumptions. In particular, it has to be assumed that the flux distribution in the sample is similar to the one in the Meissner state, where the supercurrent flows within a thin surface layer. We argue that this assumption is not unreasonable in situations that are pertinent to our measurements. In other cases, in which the magnetic induction changes throughout the sample cross-section, the calibration is only approximate.

The relaxation measurements were carried out at a temperature of 20 K, stabilised within  $\pm 10$  mK, in fields up to 20 kOe (the apparent irreversibility field at this temperature, determined both from magnetic hysteresis and  $M(T)$  measurements). These conditions, which were dictated by experimental factors, allowed us to reach an optimal stability and reproducibility of the measurements.

The interpretation of relaxation measurements is

hampered by the strong dependence of the critical current on the magnetic induction [30], which influences the result, unless the magnetic induction is approximately uniform within the sample. This can only be achieved, when small changes are imposed on a large, uniform background.

The relaxation measurements to be described in the present paper were carried out on a field-cooled (FC) sample. External fields up to 20 kOe were imposed at a temperature above  $T_c$  and the sample was cooled to  $T = 20$  K. The field registered by the Hall sensor changes only by about 20 Oe during this process in an applied field of 20 kOe, confirming that the FC samples are almost fully penetrated.

In the actual relaxation measurements the process of flux penetration is monitored after the applied field is changed by a small amount with respect to its value during cooling (fig. 2). The measure of flux penetration is the difference between the field registered by the Hall probe and the applied field, which is evidently proportional to the magnetization of the sample. Apart from the high accuracy of the Hall probe, this method requires an excellent stability of the external field. To ensure this, we have to use a superconducting magnet working in the persistent mode. Inevitably, due to the large self-inductance of the magnet, shunted by a small resistance, the response time of the system is long. In measurements

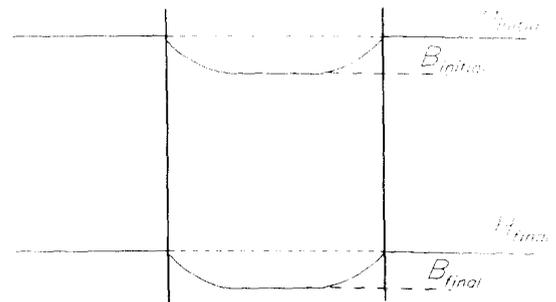


Fig. 2. Schematic flux distribution in a sample that is field-cooled (FC) to low temperatures from  $T > T_c$ . The upper part (solid line) corresponds to the initial quasi-equilibrium situation where the external field is  $H_{\text{initial}}$ , while the lower part of the figure (solid line) represents the equilibrium state, towards which relaxation occurs, after the field has been changed to the new value  $H_{\text{final}}$ . The differences  $B_{\text{initial}} - H_{\text{initial}}$  and  $B_{\text{final}} - H_{\text{final}}$  are equal to the initial FC magnetization and to the equilibrium magnetization  $M(H_{\text{final}})$ , respectively.

above  $T_c$ , the change in the external field was found to be an exponential function of time, with a time constant  $t_0$  of 6.8 s.

The sluggish change of the external field causes some problems in the interpretation of the results at short times. In case of an abrupt change of the external field from  $H_{\text{initial}}$  to  $H_{\text{final}}$ , the instantaneous response of the sample is “fully diamagnetic”, i.e.  $4\pi M(0) = -(H_{\text{final}} - H_{\text{initial}})$ , the relaxation starts immediately at  $t=0$  and is seen as a monotonic approach of  $M(t)$  to zero. In reality, the external field starts to change at  $t=0$  and reaches  $H_{\text{final}}$  after about 5 to 10 times  $t_0$ . The response of the sample follows this change, but at the same time relaxation sets in. Therefore, the early stages of relaxation cannot be monitored by measuring  $M(t)$ . In some cases the distortion is so drastic that  $|M(t)|$  is not a monotonic function of time, but goes through a maximum. However, beyond about 100 s we can consider  $4\pi M(t) = H_s - H_{\text{final}}$  as the proper relaxation function.

There is also a practical restriction on the side of the long time-scale. It appears that for  $t^* > 3000$  s we are not able to carry out a satisfactorily accurate analysis of the results of the long-time relaxation. It would require measurements for at least several hours. Table 1 contains the parameters, determined

Table 1

The experimental conditions and the fitting parameters for eq. (6) for the data presented in figs. 3–6. The details of the fitting procedure for the determination of the parameters  $p$  and  $q$  are illustrated in fig. 6(a) and (b). The crossover time is estimated in a way shown in fig. 6(c). The parameter  $\alpha$  is determined from  $q$ , characterizing the long-time relaxation. For comparison,  $\epsilon = U_c / k_B T (= 1/q)$  is computed from eq. (7)

| $H_{\text{final}}$ (kOe) | $\Delta H$ (kOe) | $p$  | $q$   | $t^*$ (s)      | $\alpha$ | $\epsilon$ |
|--------------------------|------------------|------|-------|----------------|----------|------------|
| 2.464                    | 0.574            | 0.1  | 0.15  | $1500 \pm 500$ | 0.87     | 6.7        |
| 5.964                    | 0.549            | 0.24 | 0.245 | $420 \pm 50$   | 0.80     | 4.1        |
| 8.290                    | 0.590            | 0.25 | 0.35  | $200 \pm 50$   | 0.74     | 2.9        |
| 15.74                    | 0.560            | 0.45 | 1.1   | $100 \pm 50$   | 0.48     | 0.91       |
| 21.611                   | 0.545            | –    | 1.55  | $< 150$        | 0.39     | 0.65       |
| 22.994                   | 0.532            | –    | 2.45  | $\sim 10^2$    | 0.29     | 0.41       |
| 25.189                   | 0.543            | –    | 3.0   | $\sim 10^2$    | 0.25     | 0.33       |
| 5.950                    | 0.140            | 0.27 | –     | $\gg 3000$     | –        | –          |
| 5.942                    | 0.275            | 0.24 | 0.205 | $520 \pm 100$  | 0.83     | 4.9        |
| 5.964                    | 0.549            | 0.24 | 0.245 | $420 \pm 50$   | 0.80     | 4.1        |
| 5.971                    | 1.126            | –    | 0.23  | $\sim 100$     | 0.81     | 4.3        |
| 5.968                    | 1.727            | –    | 0.23  | $< 100$        | 0.81     | 4.3        |

from fitting the data to the power-law dependences given by eqs. (6). In case there are doubts about the quality of the fitting, we do not place those parameters there.

### 3. Results and discussion

In fig. 3 the time dependence of the stray field measured at the sample surface by the Hall probe is presented in a log–log plot, for external field changes of about 550 Oe starting from different  $H_{\text{initial}}$  values, ranging from 2.5 up to 25 kOe. For low values of  $H_{\text{initial}}$ , the relaxation is slow and the remanent magnetic moment is large. It is clear that these results cannot be described by a power-law dependence over the whole time-window. From fig. 4, which gives some of the results in a semilog representation, we conclude that the logarithmic time dependence is not appropriate either for all the data. The lowest two curves in fig. 3 were registered above the irreversibility field of about 20 kOe. However, there is no qualitative difference between these data and the data obtained at lower fields, except for a much faster relaxation observed in higher fields. It would be interesting to continue these measurements to fields much higher than  $H_{\text{irrev}}$ . This, however, will require the development of an experimen-

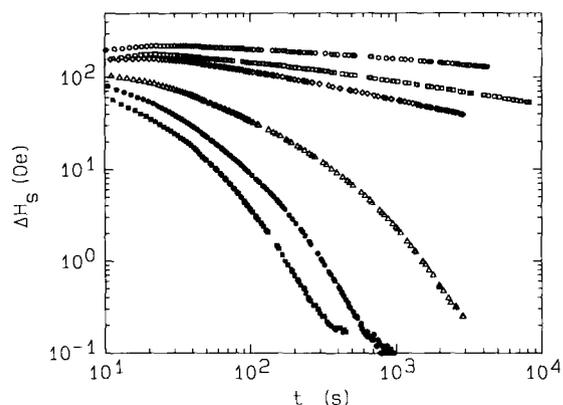


Fig. 3. Experimental data for the time evolution of the remanent magnetization (stray field  $\Delta H_s = H_s - H$ ) of a disk-shaped single-crystalline sample of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  at 20 K after a field change. Curves from top to bottom were registered after the field was decreased by about 550 Oe from the initial value to: 2.464, 5.964, 8.290, 15.74, 22.994, and 25.189 kOe, respectively.

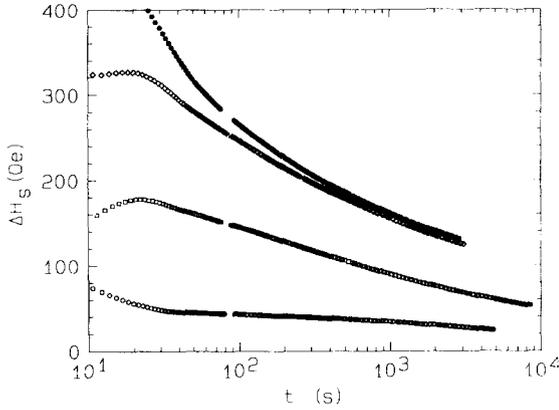


Fig. 4. Stray field  $\Delta H_s = H_s - H$  registered after the initial field was decreased to about 6 kOe, for the values of  $H_{\text{initial}} - H_{\text{final}}$  from top to bottom, 1.727, 1.126, 0.549 and 0.140 kOe.

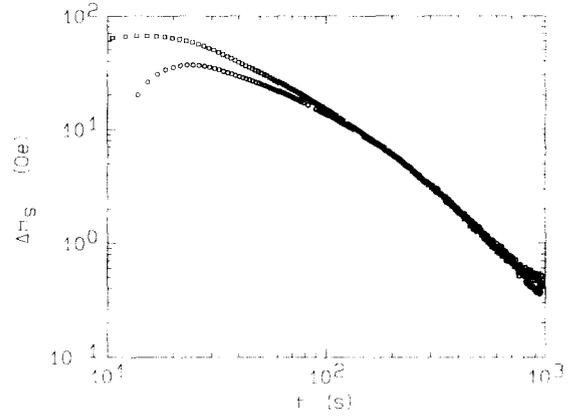


Fig. 5. Stray field  $\Delta H_s = H_s - H$  registered after the initial field was decreased to 21.6 kOe, for the values of  $H_{\text{initial}} - H_{\text{final}}$  of 0.545 kOe (squares) and 0.253 kOe (circles).

tal set-up giving access to shorter relaxation times.

The first rows of table 1 give the fitting parameters for the curves in fig. 3. A fast decrease of the crossover time  $t^*$  with increasing field is observed, accompanied by an increase of the exponents  $p$  and  $q$ . When the crossover time becomes shorter than about 100 s, we cannot carry out a reliable fitting of the short-time relaxation. We observe that  $p$  is not exceeding 0.5, but it tends to this value when  $q$  increases above 1, in agreement with the expectations based on eq. (6).

The last rows of table 1 show the fitting parameters for the curves shown in fig. 4, where the amplitude of the field change  $\Delta H$  is different but the bias field is approximately the same, for all curves. Here we do not observe a dependence of  $p$  and  $q$  on  $\Delta H$ , despite a steep decrease of  $t^*$  with increasing  $\Delta H$ . This again is in agreement with prediction summarized in eqs. (6). For the two highest values of the field change, the relaxation curves are almost identical at long times, as expected (cf. fig. 4). However, agreement with the theoretical predictions is not perfect: one should expect a coincidence of all  $M(t)$  curves, when  $t$  exceeds  $t^*$ . The coincidence is observed only for times much larger than  $t^*$ . For the data registered in a large bias field, as shown in fig. 5, the crossover time  $t^*$  is short and a very good coincidence is obtained for  $t > 100$  s when  $\Delta H$  equals 0.5 and 0.25 kOe. The fitting procedure for the determination of the parameters  $p$ ,  $q$  and  $t^*$  are illustrated in fig. 6.

There is a slight disagreement between the  $\alpha$ -values as derived from the exponents  $p$  and  $q$ . We suppose that this is connected with the approximate nature of the assumed  $I-E$  dependence. Also, eq. (6) are only approximate and the experimental situation does not correspond exactly with any of the calculations done. However, we are confident that the obtained results illustrate well the existence of the two-stage relaxation process and its dependence on  $\Delta H$  and  $H$ . Values for the exponents  $p$  and  $q$  obtained at the lowest fields are consistent with the collective creep theory [31], where they should have a value of about  $\frac{1}{2}$ . This, however, may be fortuitous.

Ries et al. [23] obtained a good description of transport and magnetization measurements on polycrystalline  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  in terms of  $J = E^{1-\alpha}$  characteristics, through eleven orders of magnitude of  $E$  changes, for a large range of magnetic fields and temperatures. A decrease of  $\alpha$  is observed when the irreversibility line is approached, either by an increase of field or temperature. This is in a qualitative agreement with our results ( $\alpha$  would correspond to  $(U_c + k_B T)/k_B T$  in eq. (6)); a decrease of  $\alpha$  is consistent with a suppression of the pinning energy  $U_c$ , while we observe a strong increase of  $q = k_B T/U_c$ . Similar conclusions are derived from the analysis of inductive measurements of the current-voltage characteristics on Bi-Sr-Ca-Cu-O ceramic rings, performed by Paul and Meier [25]. The analysis of the magnetization relaxation in single crystals by Hu

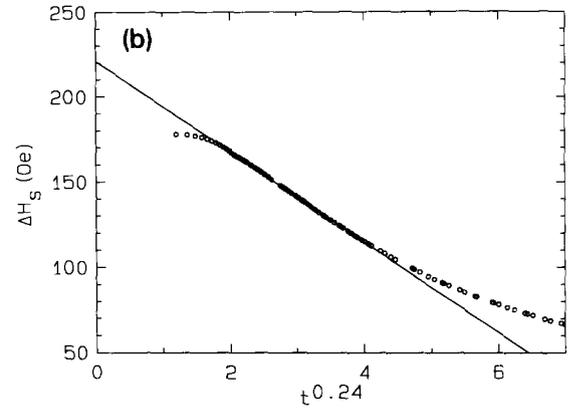
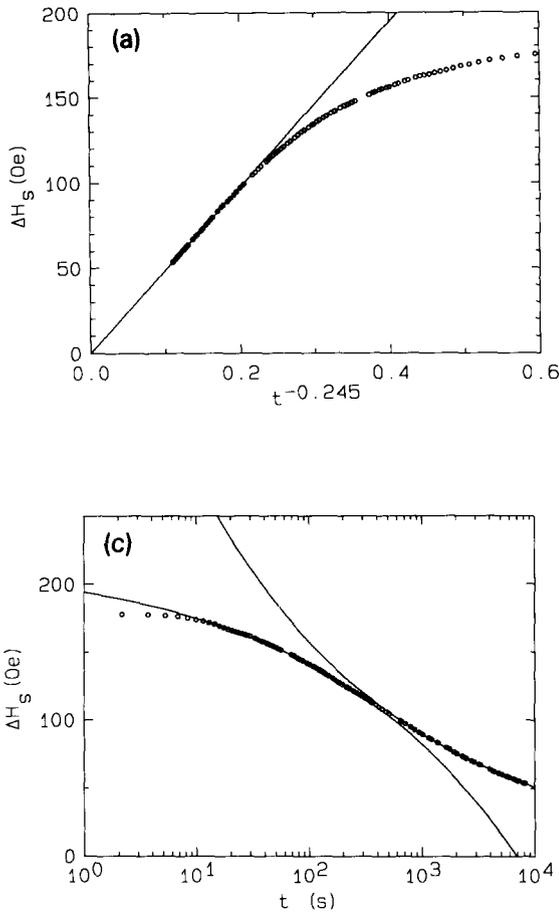


Fig. 6. Illustration of the fitting procedure to the data obtained after a decrease of the initial field with 0.549 kOe to 5.964 kOe. (a) Assuming a power-law time dependence,  $\Delta H \sim t^{-q}$ , for long times, a value for the exponent  $q$  of 0.245 is found which would allow continuation of the data to  $\Delta H=0$  for  $t \rightarrow \infty$  (solid line). (b) Assuming a power-law dependence,  $\Delta H \sim t^p$ , for short times a value for the exponent  $p$  of 0.24 is found giving a good fit in a broad time range. For  $t \rightarrow 0$  the solid line continues to  $\Delta H_S(0) = 221$  Oe. This is in good agreement with the estimated “screening efficiency” for our geometry equal to 0.39 (see text):  $\Delta H$  of 0.549 kOe multiplied by 0.39 gives 214 Oe. The deviation of experimental data from the solid line at shortest times is due to an initially slow change of the external magnetic field, as explained in the text. (c) The fitting parameters determined in (a) and (b) are used to draw the solid lines in this figure for comparison with experimental data. The crossover time  $t^*$  is found from the condition of a nearest distance between the solid curves in this figure.

[26], leads to the validity of the power-law  $J$ - $E$  dependence in a large range of  $E$ ,  $B$  and  $T$ . The “S-type” shape of the relaxation curves  $M$  versus  $\ln(t)$ , can be found in many reports on high- $T_c$  materials, as for the  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  compound [3].

The monotonic change of the parameters  $p$ ,  $q$  and  $t^*$  functions of the bias field, when the field crosses  $H_{\text{irrev}}$ , as determined in magnetization measurements, would suggest that the irreversibility line has a purely dynamical character, while the appearance of irreversible, hysteretic effects is an intrinsic property of highly nonlinear, dynamical systems. The value of  $H_{\text{irrev}}$  is often reported to depend on the time-scale of the measurement: on the times involved in DC magnetization measurements, on the frequency of AC susceptibility measurements, and on the criterion used to define  $H_{\text{irrev}}$  in resistance measurements.

The apparent relaxation rate  $S = dM/d \ln t$  is a function of the critical current (more precisely, of Bean’s penetration depth  $d_0$ ), the time and the parameter  $\alpha$  also being field- and temperature dependent. The maximum in  $S(T)$ , that has been reported for different materials, possibly results from a crossover between the temperature region where  $H < H^*$  to that where  $H > H^*$  (where  $H^*$  corresponds to the field of the first full flux penetration in the critical state model), assuming that the relaxation is measured at the same field for all temperatures – a study carefully performed by Xu et al. [32]. We note that the kink in  $S(t)$  at  $t = t^*$ , that was predicted in the original model [9] but was absent in the numerical calculations [11,12] as well as in the approximate analytical solutions [10], is not observed in our experiments.

It is possible to show on the basis of the electro-

dynamic equations that this kind of jump is forbidden for any continuous solution for the magnetic induction profile across the sample for  $t < t^*$  and  $t > t^*$  [27].

It should be pointed out that our results do not enable us to confirm the logarithmic dependence of the flux pinning barrier on the current density, given in eq. (5), or any specific field dependence of the current density, except for a strong suppression of the pinning energy and the critical current density by the magnetic field. Numerical calculations [11,12] have shown that the details of the relaxation function  $M(H, t)$  are rather weakly dependent on the specific form of the assumed  $U(j)$  function. The equivalence with a nonlinear  $E-j$  characteristic is another indication of the universality of the nonlinear flux diffusion model and indicates that the link between this phenomenological description and possible microscopic models deserves further study.

The dependence of the relaxation process on the boundary conditions is omitted in the derivation of eq. (6). It is weak, but also requires further studies, both experimental and theoretical. There are only a few, simplified situations where this effect is taken into account [31].

#### 4. Summary and conclusion

A procedure for magnetization relaxation measurements has been developed. The sample is cooled down from  $T > T_c$  in a large bias magnetic field and then relaxation is registered after a small change of the magnetic field. In this way, the effect of an inhomogeneous field distribution inside the sample volume on  $j_c$  can be avoided. We find that the results are qualitatively well described by the model of the self-organized critical state: there are short- and long-time domains of the relaxation, both characterized by a power-law time dependence. The crossover time depends on the bias field (on the critical current density) and on the amplitude of the field change. The long-time relaxation is independent of the initial field change. We find a good qualitative agreement with existing, published results: for large field changes the magnetization has power-time dependence (the “long-time” relaxation is generally observed, as the crossover time is rather short in that

case). while for small field changes (large critical current) a nearly logarithmic time dependence is observed (the crossover time is larger than the observation time). However, this “short-time” relaxation can be fitted, at least equally accurately, by a power-law dependence  $M - M(0) \sim t^n$ .

The results suggest that the decrease of the critical current and of the pinning energy with magnetic field, is responsible for the appearance of the reversible magnetization.

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