

Magnetostriction of a superconductor: Results from the critical-state model

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In many cases, the critical-state theory can be treated as a sufficiently accurate approximation for the modeling of the magnetic properties of superconductors. In the present work, the magnetostrictive hysteresis is computed for a quite general case of the modified Kim–Anderson model. The results obtained reproduce many features of the giant magnetostriction (butterfly-shaped curves) reported in the literature for measurements made on single-crystal samples of the high-temperature superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$. It is shown that the addition of a contribution to the magnetostriction in the superconducting state which is of similar origin as in the normal state, offers a broader phenomenological interpretation of the complex magnetostriction hysteresis found in such heavy-fermion compounds as UPt_3 , URu_2Si_2 , or UBe_{13} . © 1996 American Institute of Physics. [S0021-8979(96)66208-X]

I. INTRODUCTION

Measurement of the magnetization has been the most commonly used experimental technique for the study of flux pinning. Recently, however, magnetostriction studies have been shown informative. For example, Ikuta *et al.*¹ observed a giant magnetostriction in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ single crystals in the superconducting state and showed that the largest contribution to this effect comes from the interaction between the critical currents induced by changing the external field and the applied field, i.e., the Lorentz force. Much attention has also been directed to dilatometry studies of heavy-fermion superconductors. In UPt_3 ,² the irreversible contribution to the magnetostriction is small but in UBe_{13} ,³ UPd_2Al_3 ,⁴ and URu_2Si_2 ,⁵ large sample size changes are observed.

In large fields, when the magnetic induction B changes negligibly across the sample (e.g., in URu_2Si_2 , for $H \geq 10^3$ Oe,⁶ the idealized situation for a slab of thickness $2D$ may be written as $B(x) \approx H - (4\pi/c)j_c(D-x)$, with $x=0$ at the center of the slab. Here, H is the external magnetic field and j_c is the critical current density, $(4\pi/c)j_c = -dB/dx$. The local Lorentz force exerted on the current carriers, $f = (1/c)j_c B(x)$, is directed towards the center of the sample and leads to its compression, for increasing external field. This force must be compensated by the internal local stress, $\sigma(x)$; $\partial\sigma/\partial x = (1/c)j_c B(x)$. Hence, with $j_c = -(c/4\pi)dB/dx$, we find that $\sigma(x) = -[H^2 - B^2(x)]/8\pi$, for the boundary condition $\sigma(D) = 0$. For a material characterized by the elastic constant c_{11} , the relative change of the sample size, $\Delta D/D$, is given by

$$\frac{\Delta D}{D} = \frac{1}{D} \int_0^D \frac{\sigma(x)}{c_{11}} dx, \quad (1)$$

which is equal to $-1/(8\pi c_{11}D) \int_0^D [H^2 - B^2(x)] dx$. For the present case, with $(c/4\pi)j_c \ll B/D$, we have: $\Delta D/D \approx -f_p/(2c_{11})D$. Hence, $\Delta D/D$ is directly related to the flux-pinning force density, $f_p = (1/c)Bj_c$.

II. MAGNETOSTRICTION IN THE MODIFIED KIM–ANDERSON MODEL

For qualitative calculations, we shall consider the simplest possible geometry: a slab of thickness $2D$, with the external magnetic field H applied along its surface. We use a modified Kim–Anderson formula for defining the critical current density, $j_c(B)$, corresponding to a local gradient of the magnetic induction:

$$j_c = \alpha/(B+h)^n = -(c/4\pi)dB/dx. \quad (2)$$

This dependence unifies many forms of the critical state models. The calculating the magnetization for this geometry and for $j_c(B)$ given by Eq. (2) can be found in Ref. 7. This previous work⁷ contains an analysis of other effects observed in superconductors which can be explained using the model of a critical state. The present work provides additional explanation of these phenomena and may be considered as a continuation of our work on this subject.

The calculation of the magnetostriction consists of: (i) finding the magnetic field distribution during the magnetization process, (ii) integrating the equation $\partial\sigma/\partial x = (1/c)j_c B(x)$ with the boundary condition that $\sigma(D) = 0$, and (iii) using Eq. (1) for calculation of the magnetostriction. The solution of Eq. (2) determines the flux distribution and has the form: $B(x) = [\pm (4\pi/c)\alpha(n+1)x + \beta]^{1/(n+1)} - h$, where the integration constant β is determined from the boundary condition $B(\pm D) = H$, with the sign of $dB(x)/dx$ dependent on the most recent field change direction occurring at point x . For the virgin magnetization curve, β is given by: $\beta = (H+h)^{n+1} - H_m$, where $H_m = (H^*+h)^{n+1} - h^{n+1}$, and H^* is the field of the first full flux penetration into the center of the slab, $H^* = [\alpha(n+1)D + h^{n+1}]^{1/(n+1)}$. Some examples of the flux distribution for different situations are drawn schematically in Fig. 1. In this figure, B_0 is the magnetic induction in the sample center and is given in Table I for various stages of the magnetization process. The magnetostriction during the virgin magnetization process, for $H < H^*$, is found to be

$$\frac{8\pi c_{11}\Delta D(H)}{D} = H^2 - \frac{1}{H_m} \left\{ h^2[(H+h)^{n+1} - h^{n+1}] \right\}$$

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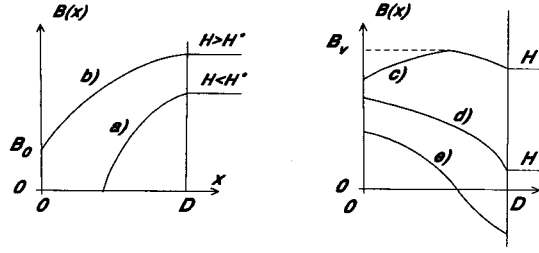


FIG. 1. The magnetic induction distribution in a sample volume when (a) field increases from 0 to $H < H^*$, (b) field increases to $H > H^*$, (c) and (d) when field decreases from H_{\max} to H , and (e) when field decreases below 0. B_0 is the field at the slab center. B_v is the field at the kink of the flux distribution at x_v , occurring for a certain range of H values after the sweep direction of H is changed. x_v is equal to $1 - [(H_{\max} + h)^{n+1} - (H + h)^{n+1}]/(2H_m)$. B_v and B_0 are defined in Table I.

$$+ (n+1) \left[\frac{(H+h)^{n+3} - h^{n+3}}{n+3} - 2h \frac{(H+h)^{n+2} - h^{n+2}}{n+2} \right]. \quad (3)$$

In order to provide the results for the entire magnetization loop in a transparent form, we define the following quantities:

$$\begin{aligned} F_{i,1}(H_{i,-}, H_{i,+}) &= h^2 [(H_{i,+} + h)^{n+1} - (H_{i,-} + h)^{n+1}] / H_m, \\ F_{i,2}(H_{i,-}, H_{i,+}) &= -(n+1)/(n+2) 2h [(H_{i,+} + h)^{n+2} - (H_{i,-} + h)^{n+2}] / H_m, \\ F_{i,3}(H_{i,-}, H_{i,+}) &= (n+1)/(n+3) [(H_{i,+} + h)^{n+3} - (H_{i,-} + h)^{n+3}] / H_m, \end{aligned}$$

with $H_{i,-}$ and $H_{i,+}$ defined in Table I, for situations corresponding to those in Fig. 1. The index i enumerates different branches of the magnetic induction curves of continuous $j_c(x)$ dependence. Using the above equations, the magnetostriction can be written as

$$8\pi c_{11} \Delta D(H)/D = H^2 - \sum_{i=1,2} f(H_{i,-}, H_{i,+}), \quad (4)$$

where

$$f(H_{i,-}, H_{i,+}) = S(H_{i,-}, H_{i,+}) [F_1(H_{i,-}, H_{i,+})$$

TABLE I. Characteristic fields corresponding to the curves (a) to (e) in Fig. 1. B_v is equal to $\{0.5[(H_{\max} + h)^{n+1} + (H + h)^{n+1}]\}^{1/(n+1)} - h$.

Case	$H_{0,-}$	$H_{0,+}$	$H_{1,-}$	$H_{1,+}$
a	0	H	0	0
b	$B_0 = [(H+h)^{n+1} - H_m]^{1/(n+1)} - h$	H	0	0
c	$B_0 = [(H_{\max} + h)^{n+1} - H_m]^{1/(n+1)} - h$	B_v	B_v	H
d	$B_0 = [(H+h)^{n+1} + H_m]^{1/(n+1)} - h$	H	0	0
e	$B_0 = [-(H + h)^{n+1} + (H^* + h)^{n+1} + h^{n+1}]^{1/(n+1)}$	0	0	H

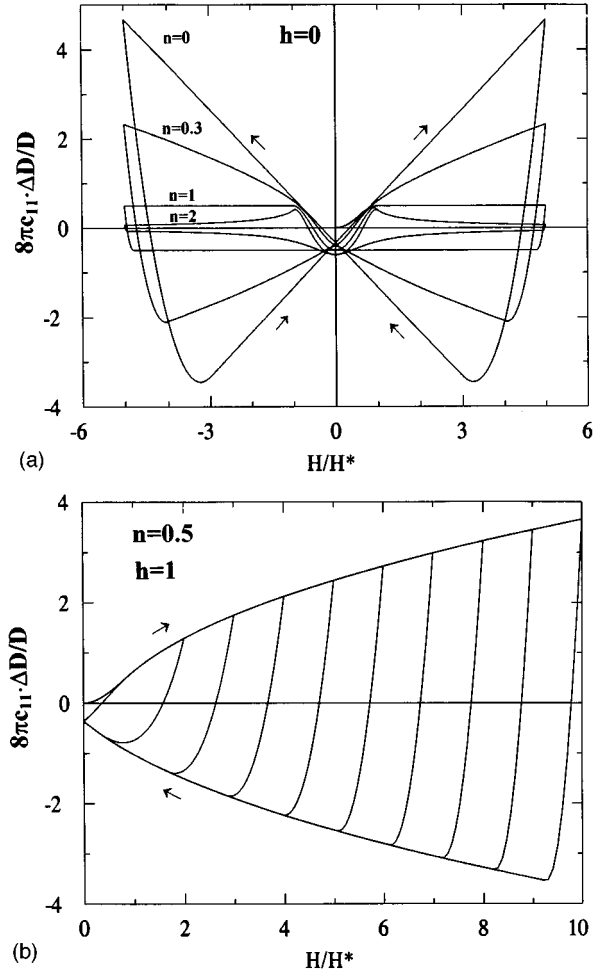


FIG. 2. The calculated magnetostriction curves, the virgin one and the entire hysteresis loop; (a) $h=0$, while n changes from 0 (the Bean model) to $n=2$, (b) when $n=0.5$, $h=1$ while H_{\max} ranges between 2 and 10. The arrows indicate the direction of field change.

$$+ F_2(H_{i,-}, H_{i,+}) + F_3(H_{i,-}, H_{i,+})]. \quad (5)$$

The function $S(H_{i,-}, H_{i,+})$ in Eq. (5) is equal to $+1$ when $H_{i,-} \leq H_{i,+}$ and to -1 , otherwise. The remaining part of the magnetostriction hysteresis curve, not covered in the Table I, may be computed using symmetry: $\Delta D(-H) = \Delta D(H)$.

Some examples of the calculated magnetostriction hysteresis curves are shown in Fig. 2. A large variety of $\Delta D(H)$ dependences is found: from $\Delta D(H)$ proportional to H at $H > H^*$, when $n=0$ (Bean model), to $\Delta D(H)$ strongly suppressed by field when $n > 1$. For $n > 0$, an abrupt passing through $\Delta D(H) = 0$ is found, when the field sweep direction changes. The field width of this transition region decreases strongly with an increase of the maximum field applied [Fig. 2(b)]. This is due to a low value of the critical current density at large fields and a decrease in the field change required to reverse the direction of current flow in the sample volume. A significant remanent magnetostriction due to a frozen-in magnetic field is observed. The features discussed above are commonly observed for high- T_c and heavy-fermion superconductors.

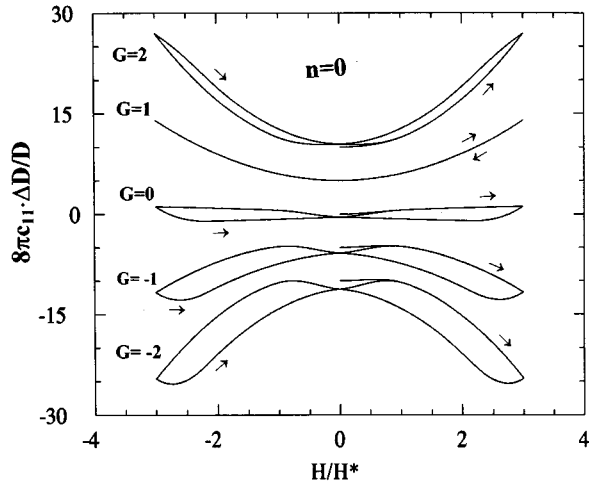


FIG. 3. The calculated magnetostriction curves for $n=0$ (the Bean model) for several values of the parameter G , determining the normal-state contribution to the magnetostriction: $G=2, 1, 0, -1$, and -2 , for curves from top to bottom, respectively. One must note that the values of G depend on the units of the magnetic field assumed, which are here equal to H^* . The vertical scale has been shifted for some curves, for better clarity. The arrows indicate the direction of field change.

An unusual effect has been observed recently⁵ for single crystal of URu_2Si_2 . For fields near the upper critical field, $H_{c2}(T)$, the irreversible contribution to the magnetostriction changes sign compared to the sign observed at lower fields. Moreover, it was observed⁵ that the sign of the irreversible contribution to the magnetostriction for some experimental configurations disagreed with the prediction of the critical-state model. On the other hand, a large background contribution to the magnetostriction, $\Delta D/D = bB^2$, with the parameter b of the order of $10^{-16}/G^2$, has been reported for heavy-fermion materials,^{2,3,5} which is apparently of the same origin as the normal-state magnetostriction. The role of this contribution on the overall shape of the magnetostriction may be considered. We note that the critical-state magnetostriction component results from the calculation of an integral over the $B^2(x)$ dependence. The total magnetostriction in the superconducting state is then assumed to be the sum of the critical state and the normal-state-like components. The sign of both contributions must be considered carefully. The critical-state magnetostriction depends on the sign of the magnetic field and current while the normal-state contribution depends on the sign of the coefficient b . Using these arguments, we may write

$$8\pi c_{11}\Delta D(H)/D = H^2 - \sum_{i=1,2} [f(H_{i,-}, H_{i,+}) - G|f(H_{i,-}, H_{i,+})|], \quad (6)$$

with $f(H_{i,-}, H_{i,+})$ given by Eq. (5) and $G = 8\pi c_{11}b$. Magnetostriction calculated from Eq. (6) for a few values of the parameter G is shown in Fig. 3. An important feature of these results is that both signs of the magnetostriction and both

signs of the irreversible contribution to the magnetostriction hysteresis are possible, while the shape of the curves might only be weakly affected by the presence of a normal-state contribution (except for the presence of a large background proportional to H^2). The hysteresis of the normal-state-like contribution to the magnetostriction in the superconducting state is due to inhomogeneity of the magnetic induction across the sample volume, which is caused by the presence of critical current. It is easy to show that at large fields, in particular, near the upper critical field, both contributions to the irreversible magnetostriction are proportional to the critical current density. Hence, the most intriguing property of the magnetostriction in some heavy-fermion superconductors, i.e., the change of sign of the irreversible part of the magnetostriction as a function of the applied field, cannot be explained by this model, if it is assumed that the parameter G in Eq. (6) is independent of the magnetic field value.

Using Eq. (5), magnetic measurements⁶ in conjunction with the value of the elastic constant c_{11} allow for a determination of $\Delta D/D$. In URu_2Si_2 , $c_{11} = 2.4 \times 10^{12}$ erg/cm³,⁸ yields $2\Delta D/D = 3.6 \times 10^{-8}$ and $2\Delta D/D = 6 \times 10^{-8}$, for H/c and H/a , respectively, in reasonable agreement with measured properties.⁵ On the other hand, for UPt_3 , the elastic constant is larger, about 6×10^{12} erg/cm³,⁹ and the critical current density is much lower,⁷ resulting in an irreversible contribution to the magnetostriction which is hardly observable experimentally.

Gloos *et al.*⁴ reported significant difference between the thermal-expansion curves measured during cooling in a magnetic field through the superconducting transition and those measured during heating with the same magnetic field applied at low temperature, for the heavy-fermion superconductor UPd_2Al_3 . Any hysteresis of this kind finds its phenomenological explanation within the critical state model and is of the same nature as the magnetization hysteresis in $M_{FC}(T)$ and in $M_{ZFC}(T)$ measurements, as described by us in detail in Ref. 7.

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