Nonlinear flux diffusion and ac susceptibility of superconductors: Exact numerical results

Z. Kozioł[a)] and R. A. Dunlap
Department of Physics, Dalhousie University, Halifax, N.S., Canada B3H 3J5

The ac response of a slab of material with electrodynamic characteristics $E \sim j^{\kappa+1}$, $\kappa \geqslant 0$, is studied numerically. From the solutions of the nonlinear diffusion equation, the fundamental and higher-order components of the harmonic susceptibility are obtained. A large portion of the data for every κ can be scaled by a single parameter, $\xi \equiv t^{1/(\kappa+2)}\cdot H_0^{\kappa/(\kappa+2)}/D$, where t is the period of the ac field at the surface, H_0 is its amplitude, and D is the slab thickness. This is, however, only an approximate scaling property: The field penetration into a nonlinear medium is a more complex phenomenon than in the linear case. In particular, the susceptibility values are not uniquely defined by a set of only two parameters, such as κ and ξ, while one parameter, i.e., $t^{1/2}/D$, is sufficient to describe the electrodynamic response of a linear medium. [S0021-8979(96)00408-X]

I. INTRODUCTION

The problem of nonlinear diffusion has recently attracted considerable attention in diverse fields of science. One example, which is considered in the present work, deals with the magnetization process of superconductors. In the case when the response of a superconductor to an applied ac field, $H=H_0\exp(i\omega t)$, is linear, the electrodynamic properties in the superconducting state can be described in terms of the complex conductivity, $\sigma=\sigma_r+i\sigma_i$. In the flux-flow regime of high-T_c superconductors, which occurs in a broad $H-T$ range, it is usually justified to neglect the imaginary component of the complex conductivity. In that case, Maxwell's equations, $\nabla\times\mathbf{B}=4\pi/c\cdot\mathbf{j}$ and $c\nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t$, provide the following result for the field penetration into a material filling half-space, $x>0$: $B=H_0\ \eta(x)\exp(i\omega t)$, with $\eta(x)=\exp(-\lambda x)$ and $\lambda^2=2i/\delta^2$, where $\delta^2=c^2/(2\pi\omega\sigma_r)$. For a thin plate of thickness 2D, with the ac field parallel to the large surface of the plate, the following equations hold:[1]

$$4\pi\chi' = -1+\frac{1}{a}\frac{\sinh(a)+\sin(a)}{\cosh(a)+\cos(a)},$$
$$4\pi\chi'' = \frac{1}{a}\frac{\sin(a)-\sinh(a)}{\cosh(a)+\cos(a)}, \qquad (1)$$

where $a=2D/\delta$. A maximum in χ'' results when $\delta(\sigma_r,\omega)$ is comparable to the sample size. Experimental results rarely show susceptibility curves which correspond to the flux-flow result of ohmic-like behavior. Rather, the effects are most often nonlinear. The dependence of the measured susceptibility curves on the excitation current and higher-order components in the harmonic susceptibilities are observed. In the limiting case of very strongly nonlinear response, the critical-state model may be used for the calculation of the ac susceptibility. Then, only one parameter is needed to construct the hysteresis curve for the magnetization, the field of the first full penetration to the sample center, H^*. It is assumed that the harmonic susceptibility components, χ'_m and χ''_m, are defined as Fourier components of the time-dependent magnetic hysteresis curve, $M(t)/D=\sum_m\chi'_m\cos(m\,\omega t)+\chi''_m\sin(m\,\omega t)$, where m is an integer. When, for instance, $H_0<H^*$, $4\pi\chi'_1=-(1-H_0/2H')$ is obtained, then $4\pi\chi'_m=0$ for every odd $m>1$, and $4\pi\chi''_m=2H_0/3\pi mH^*$ for all odd $m>0$. To deal with situations which are more relevant to the description of real experimental results, it is necessary to investigate a nonlinear theory of the magnetic response which would bridge the two limiting cases observed: the linear response and the critical-state one. A fruitful approach to this problem is based on studies of the electrodynamic response of a medium characterized by a power-law current-voltage dependence, $j=\sigma(E)E=\sigma_0E_0(E/E_0)^{1/(\kappa+1)}$, where $\kappa\geqslant 0$. Using Maxwell's equations, the nonlinear diffusion equation describing the penetration of fields into a slab of thickness 2D lying in the yz plane[1–4] can be derived as

$$\frac{\partial\beta}{\partial\bar{t}}=\frac{\partial}{\partial\bar{x}}\left(\frac{\partial\beta}{\partial\bar{x}}\left|\frac{\partial\beta}{\partial\bar{x}}\right|^{\kappa}\right),\quad \bar{t}=\frac{t}{\tau_0},\quad \bar{x}=\frac{x}{x_0}, \qquad (2)$$

where $\beta=B/E_0$, $x_0=c/(4\pi\sigma_0)$, and $\tau_0=1/(4\pi\sigma_0)$. Recently, studies of solutions of Eq. (2) have been carried out by many authors. The exact analytical description of the response of a superconductor to an abrupt change of external field[1,2] has been compared with the results of nonlogarithmic magnetization relaxation measurements on high-T_c materials.[5] Various aspects of the ac response of superconductors has been studied as well by Dorogovtsev[6] and van der Beek *et al.*[7] A vector generalization of the critical state model has been proposed by Mayergoyz.[8,9] Recent results of Gilchrist and Dombre[10] can be compared with numerical results described in the present work. The distinctive feature of solutions of Eq. (2) is that the flux-profile penetration resembles that in the models of the critical state. When a field change is applied, the profile of perturbation spreads out from the surface towards the sample center but a region exists where the field distribution is unchanged inside. If the response to a field change of H_0 is considered, the time after the front of the field change arrives to the center t^* is given by

[a)]Electronic mail: zkoziol@is.dal.ca; http://is.dal.ca/~zkoziol/zkoziol.html

$$\frac{t^*}{\tau_0} = \frac{\kappa}{2(\kappa+1)} \frac{1}{(\kappa+2)} \left[\frac{\Gamma(1/\kappa+1)\Gamma(3/2)}{\Gamma(1/\kappa+3/2)} \right]^{\kappa} \left[\frac{\beta_0}{H_0} \right]^{\kappa} \left(\frac{D}{x_0} \right)^{\kappa+2}. \quad (3)$$

The initial magnetization, at $t < t^*$, is given by

$$4\pi M = -H_0 \left(1 - \left(\frac{t}{t^*} \right)^{1/(\kappa+2)} \frac{\Gamma(1/\kappa+3/2)}{\Gamma(1/\kappa+2)\Gamma(1/2)} \right). \quad (4)$$

Equations (3) and (4) imply that a single parameter, $\xi \equiv t^{1/(\kappa+2)} H_0^{\kappa/(\kappa+2)}/D$, can parametrize the short-time magnetization relaxation. It is informative to determine the extent to which this scaling relation is valid with respect to the ac susceptibility (with the replacement of t and H_0 by the ac field period and the field amplitude, respectively).

II. NUMERICAL MODELING OF NONLINEAR DIFFUSION

Most of the calculations of the nonlinear diffusion process presented here have been performed on an array of dimension 50×200, containing magnetic induction values B at 50 time intervals and 200 space intervals.[11] The magnetic field at the surfaces of the sample, $H_0 \sin(\omega t)$, determines the boundary conditions. An average magnetic field $\langle B(t) \rangle$ in the sample has been computed from the magnetic field distribution, every 50 time steps. Next, a Fourier time analysis of $\langle B(t) \rangle$ has been performed and the coefficients of the fundamental and higher-order terms of the harmonic content have been found. The method of computation and its results have been carefully tested. First, the magnetization relaxation process after an abrupt change of the external field has been simulated and numerical results were compared with the known exact analytical expressions derived by Kozioł and de Châtel.[2] Then, the validity of the modeling of the ac response in the limit of linear diffusion on the ac susceptibility, as given by Eq. (1), was checked. It was confirmed also that the ac susceptibility converges towards the critical-state results for large κ. The time range for which the response to the ac field becomes periodic (the initial response at short time does not satisfies this condition) was also investigated. In most cases it is safe to analyze the data taken after the initial 50 000 steps in time evolution (this time depends on κ and H_0). An additional, more reliable, criterion of stable periodicity is based on the criterion that the dc or second harmonic components are not found. At large values of H_0, the calculations become unstable abruptly. It is possible to overcome this difficulty but at significant expense in computation time (the computation of one susceptibility point requires an average of about 3 h on an IBM-PC computer 486DX2-33 MHz). Therefore, we have concentrated on performing calculations for a larger number of points at lower fields.

III. RESULTS AND DISCUSSION

The penetration of an alternating field resembles, in some ways, the response to an abrupt change of external field; the amplitude of field changes diminishes gradually in the material and, if the field amplitude at the surface is not too large, there is no penetration to a volume separated by a certain distance from the sample surface. Whether the front of the flux profile in ac penetration propagates towards the center or not, is not an easy question to answer, since the initial very slow propagation which is observed might only be due to unstable initial conditions. Within the accuracy of calculations, the flux profile has a self-replicating shape of diminishing amplitude, with perfect periodicity in time at every point in space but with a phase shift which changes with the distance from the surface. This observation is consistent with the exact results found by Mayergoyz[8] for penetration of circularly polarized electromagnetic fields. The profiles obtained for one value of an ac field amplitude coincide with the profiles computed for another ac field amplitude, if the phase lag and spatial coordinates are shifted properly.

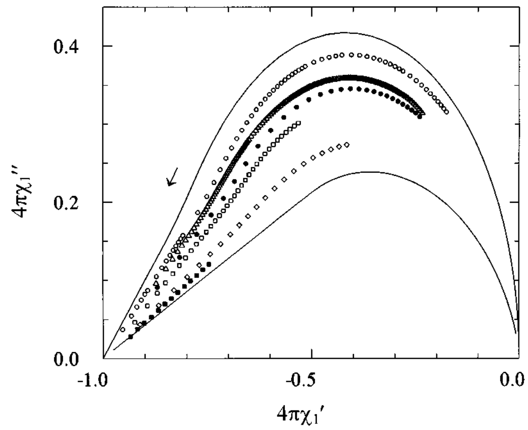

FIG. 1. The χ'' vs χ' plots of the ac susceptibility for different values of the nonlinearity parameter κ and different periods of the ac field t, (κ, t): (0.667, 25 000, ○), (0.667, 5000, △), (2, 12 500, ●), (2, 5000, □), (3, 10 000, ◇), (12, 6250, ■). Solid lines represent the critical-state and the linear-response limits for a thin plate. The difference between the data for $\kappa=2$ obtained for two different frequencies of the ac field should be noted. The direction of decreasing penetration depth is shown by the arrow.

Plots of χ'' vs χ' shown in Figs. 1 and 2 for different values of the nonlinearity parameter κ converge to the limit of linear diffusion for $\kappa \to 0$ and to the limit given by the critical-state model for large $\kappa \gg 1$. An important feature of the present results is seen in Fig. 1; susceptibility points computed for different frequencies of the ac field but the same value of κ, do not fall on the same curve. This is different from what it might be expected and seems to have been unnoticed in previous work.[10] In Fig. 3, we show that a simple scaling of the susceptibility with the amplitude of the ac field holds for the data obtained in the range of incomplete flux penetration, $\chi \sim H_0^{\kappa/(\kappa+2)}$. In Fig. 4, the real component of the first-harmonic susceptibility is drawn as a function of $H_0^{\kappa/(\kappa+2)}\, t^{1/(\kappa+2)}/D$, for different values of the field amplitude H_0, period of the field and for a few sample sizes. This latter scaling method is not perfect; small differences in the slopes of the data computed for various frequencies is found. This effect may be explained by the fact that flux profiles have a shape which depends on the time of field penetration.

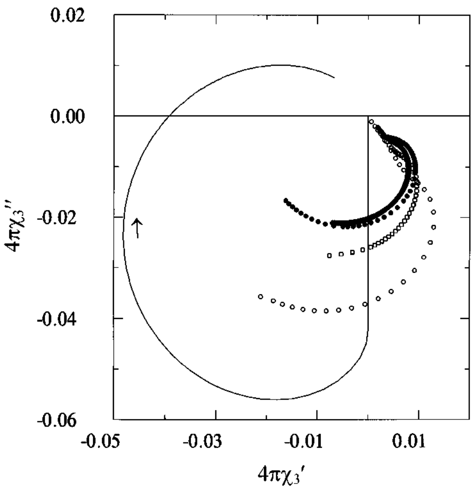

FIG. 2. The third-harmonic susceptibility χ_3'' vs χ_3' compared to the critical-state result represented by the solid line, for the following nonlinearity parameter κ and periods of the ac field t, (κ, t): (0.667, 25 000, ●), (0.667, 5000, ♦), (1, 5000, □), and (2, 12 500, ○). The direction of increasing amplitude of H_0 is shown by the arrow.

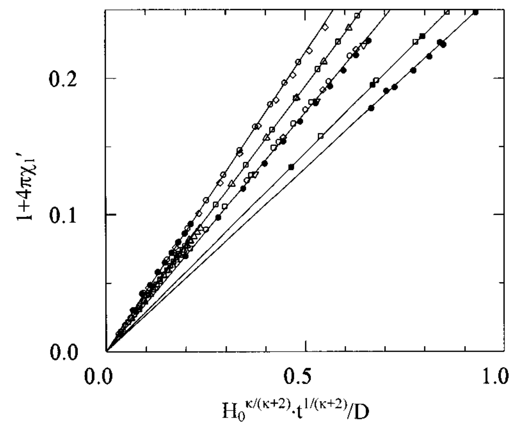

FIG. 4. Scaling of the flux penetration ($4\pi\chi_1' + 1$) by a function $H_0^{\kappa/(\kappa+2)}\, t^{1/(\kappa+2)}/D$, where H_0 gives the field amplitude, t gives the period, and D gives the sample thickness. Each of the solid lines passes through the data corresponding to the following values of the nonlinearity parameter κ: 0.667, 1, 2, 3, and 5, for lines with the smallest to largest slope. D is equal to 20 or 100 (there is no distinction between the symbols of the data corresponding to different values of D), while t is 2500 (○), 5000 (□), 6250 (◇), 10 000 (△), 12 500 (▽), 25 000 (●), and 100 000 (■).

One should expect that the parameter $\xi = t^{1/(\kappa+2)} H_0^{\kappa/(\kappa+2)}/D$ will become an exact scaling variable only for the cases when all the parameters, t, H_0, and D, are simultaneously scaled by a constant λ in the following way: $D \rightarrow \lambda D$, $H_0 \rightarrow (\lambda H_0)^{\kappa/(\kappa+2)}$, and $t \rightarrow (\lambda t)^{1/(\kappa+2)}$.

IV. CONCLUSIONS

When the ac magnetic field does not penetrate to the sample center, the magnetic susceptibility is well described by a simple scaling relation: $\chi \sim \xi$, with $\xi = t^{1/(\kappa+2)}\, H^{\kappa/(\kappa+2)}/D$. For κ values close to 0, the overall dependence of $\chi''(\chi')$ closely resembles the dependence observed in the linear case. The conductivity, however, computed from $\chi''(\chi')$ data by using the assumption that the linear theory holds, will lead to false information and yield strongly overestimated values. An experimental criterion for detecting nonlinearity would be the observation of the amplitude dependence or the existence of higher order harmonics in the ac response. The susceptibility values of a nonlinear medium are not uniquely defined by a set of two parameters only, such as κ and ξ. For experimental purposes, however, treating ξ as a scaling variable offers a sufficiently accurate method of testing for nonlinear properties of materials.

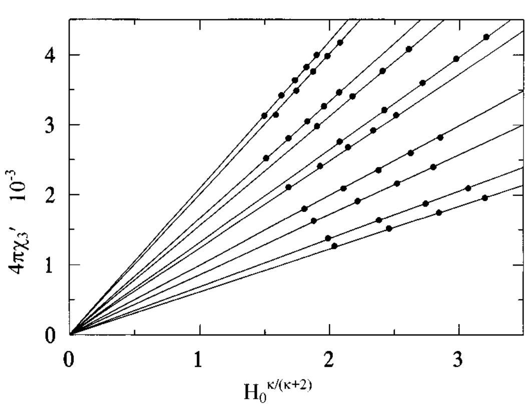

FIG. 3. The real component of the third-harmonic susceptibility as a function of the amplitude of the ac field, computed for an ac field of period equal to 25 000. The slope of solid lines decreases for increasing values of κ, which are the following: 0.667, 0.8, 1.2, 1.333, 1.667, 1.8, 2.333, 2.7, 3.35, 3.7. Similar scaling property is observed for the imaginary component of the third-harmonic susceptibility and for higher-order components as well.

ACKNOWLEDGMENTS

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